Asset-Liability Management 2025 Project - Final Version

Introduction

The project is designed to be solved in groups of 2 students. The technical work should result in a report of no more than 10 pages. The report should be in such a form that a non-mathematical person gets an idea of what you have done and why. Please place important results, tables and graphs into your report, to make it accessible to a reader who does not want to dive into programs. Use whatever programming tool comes natural for you.

You may write your report in English or Portuguese.

You may contact me with questions at the address tiagofardilha@iseg.ulisboa.pt.

The deadline for delivery of the report and underlying workings is the 1st December 2025.

Have fun!

1.Interest rate immunization

Today is 31.12.2024. Assume you have modelled the stochastic term structure development using a Cox-Ingersoll-Ross model with parameters below. Today's short rate is 1.55 %.

Maturity	rate
a	20 %
b	3.1 %
sigma	3.0 %

1.1 Calculate today's yield curve with continuous compounding for maturities up to 15 years.

Assume bonds are available for every maturity, with coupons of 3% paid at the last day of the year and a face value of €100. You want to buy bonds to support a 15-year indexed annuity starting 31.12.2025. The first amount payable on 31.12.2025 is €1,000,000. In 2025-2038, the amount payable each 31.12 is increased with €50,000 per year.

1.2 Determine a portfolio of 15 bonds that will match the liability cash flow.

1.3 Determine a portfolio of three bonds that match the present value, duration and convexity of the liability cash flow. Negative holdings are not allowed.

1.4 Create three 'buckets' of bonds: short (maturity 1-5 years), medium (maturity 6-10 years) and long (maturity 11-15 years). Each bucket contains one piece of each available bond in the specified maturity range. Determine the mix of the three buckets that matches the present value, duration and convexity of the liability cash flow.

2. Liability valuation

Simulate 1000 replications of the short rate on 31.12.2021 (one year ahead) using the CIR model.

2.1 For the three-bond portfolio you found in 1.3, calculate the present value of the annuity, the asset cash flow, and the surplus, as they will be just before the first coupon payments are received and the first annuity payment is due. Calculate the simulated Value at Risk (VaR) and Tail Value at Risk (TailVaR) of surplus on 31.12.2024 (just before payments), at 5% confidence level.

2.2 For the mix of buckets you found in 1.4, calculate the present value of the annuity, the asset cash flow, and the surplus, as they will be just before the first coupon payments are received and the first annuity payment is due. Calculate the simulated Value at Risk (VaR) and Tail Value at Risk (TailVaR) of surplus on 31.12.2024 (just before payments), at 5% confidence level.

Discuss why the VaR and TailVaR of surplus in 2.2 are lower than in 2.1.

3. Mean-Variance analysis

Your investment mandate is restricted to three asset classes: Equity, Bonds and Money market. Use the following set of assumptions for the yearly asset returns:

Asset	Asset class	E(return)	SD(return)	Correlations	1	2	3
1	Equity	0,06	0,08	**1**	1	0,016	0,158
2	Bonds	0,015	0,005	**2**	0,016	1	0,677
3	Money Market	0,005	0,002	**3**	0,158	0,677	1

Assume the risk-free rate is 0.1%.

3.1 Use the mean-variance structure to determine, with a one-year time horizon:

1. $\mathbf{w}_{min}$: The minimum asset variance portfolio,
2. $\mathbf{w}_{ref}$: The reference portfolio of risky assets for a desired return of 4%,
3. $\mathbf{w}_{tan}$: The tangency portfolio when the risk-free return is 0.1%.
4. The optimal portfolio with a risk-free asset.